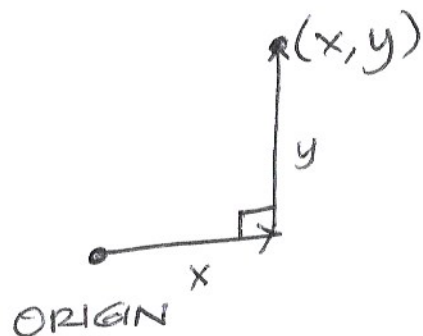


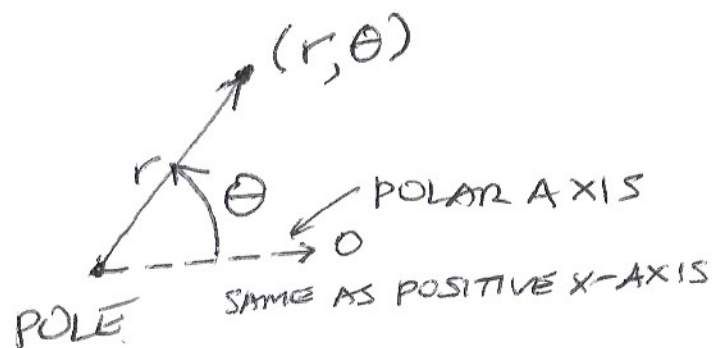
10.7 POLAR COORDINATES

RECTANGULAR COORDINATES

(x, y)



POLAR COORDINATES

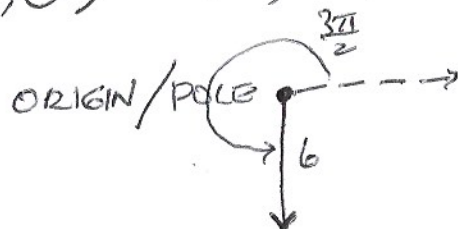


SAME POINT

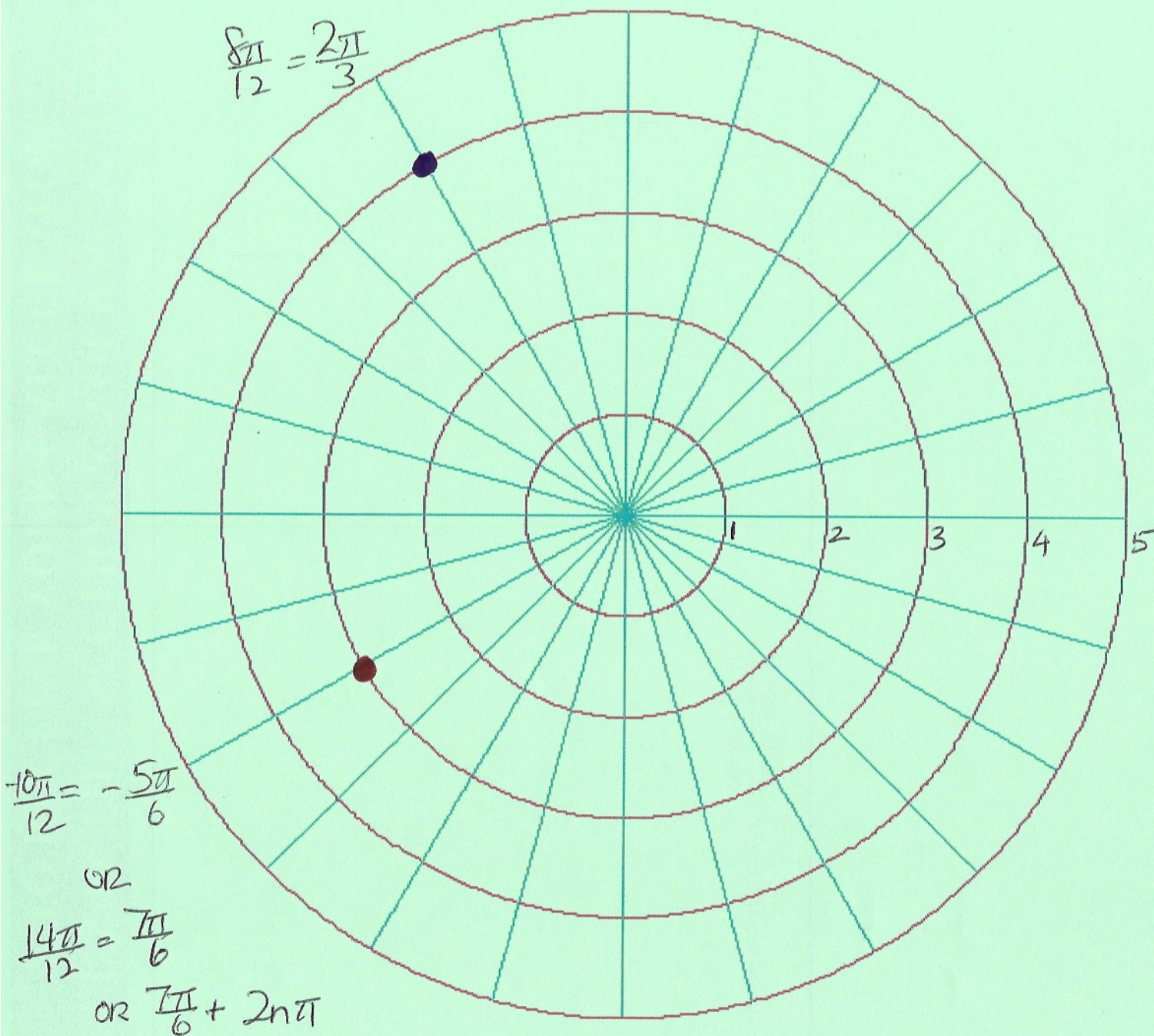
IF $(x, y) = (-5, 0)$ THEN $(r, \theta) = (5, \pi)$



IF $(r, \theta) = (6, \frac{3\pi}{2})$ THEN $(x, y) = (0, -6)$



QUADRANTAL
ANGLES
(LIES ON
X- OR y-AXIS)



- PLOT ~~THE~~
- $(r, \theta) = (4, \frac{2\pi}{3})$
 $= (4, \frac{8\pi}{12})$
 - $(r, \theta) = (3, -\frac{5\pi}{6})$
 $= (3, -\frac{10\pi}{12})$
 - $(r, \theta) = (3, \frac{7\pi}{6})$
 IS THE SAME
 POINT
 SINCE
 $-\frac{5\pi}{6}$ AND $\frac{7\pi}{6}$ ARE
 CO-TERMINAL

IN FACT,
 (r, θ) AND
 $(r, \theta + 2n\pi)$
 WHERE $n \in \mathbb{Z}$
 (IE, n IS ANY
 INTEGER)
 ARE THE SAME
 POINT

POLAR COORDINATES
FOR P

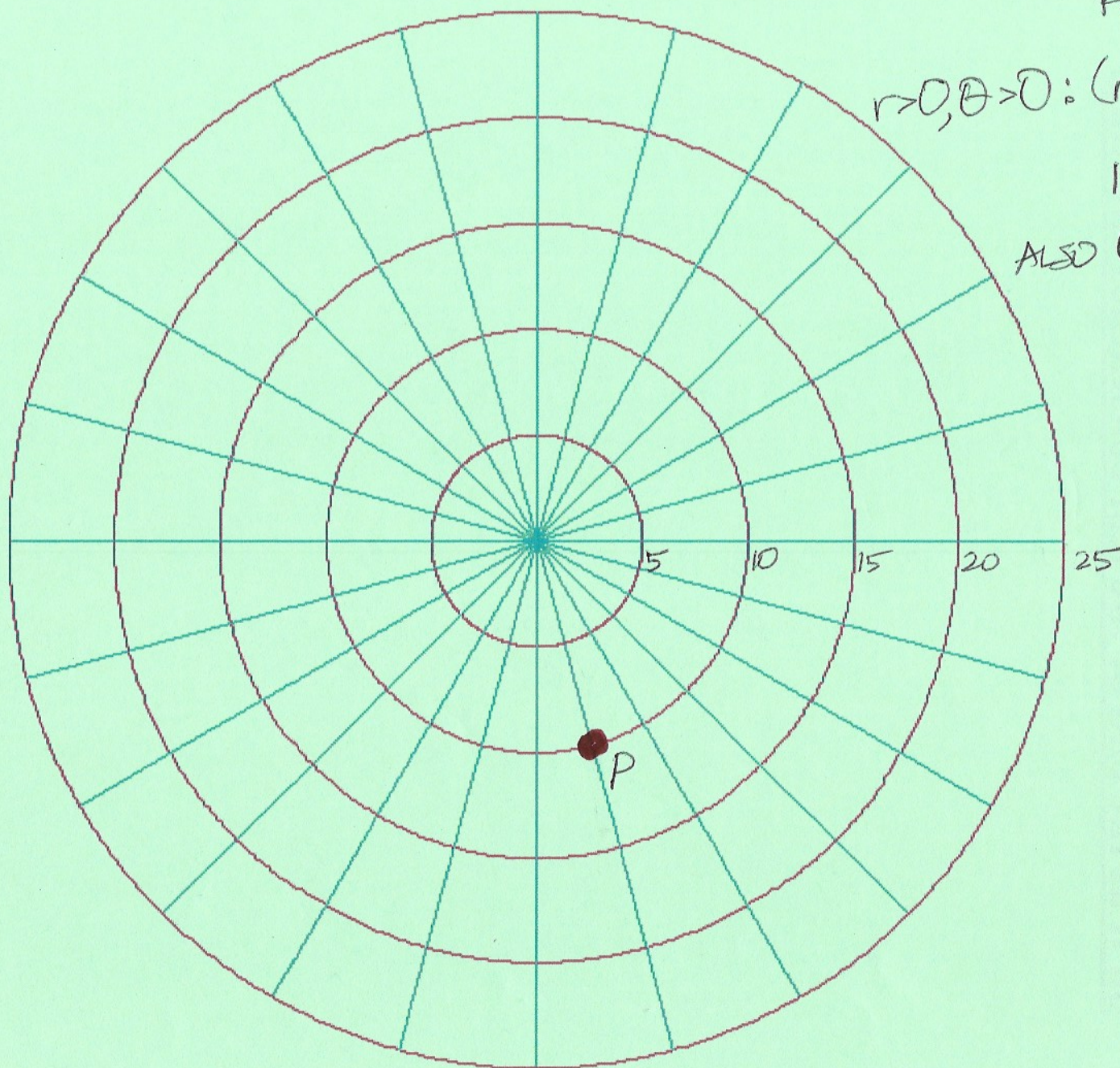
$$r > 0, \theta > 0: (r, \theta) = \left(10, \frac{19\pi}{12}\right)$$

$$\text{IF } r > 0, \theta > 0$$

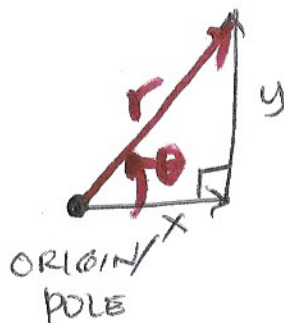
$$\text{ALSO } (r, \theta) = \left(10, \frac{19\pi}{12} + 2\pi\right) \\ = \left(10, \frac{43\pi}{12}\right)$$

$$\text{IF } r > 0, \theta < 0$$

$$(r, \theta) = \left(10, \frac{19\pi}{12} - 2\pi\right) \\ = \left(10, -\frac{5\pi}{12}\right)$$



TO CONVERT POLAR COORDINATES (r, θ)
TO RECTANGULAR COORDINATES (x, y)



GIVEN (r, θ)

$$\frac{x}{r} = \cos \theta$$

$$\frac{y}{r} = \sin \theta$$



$$x = r \cos \theta$$

$$y = r \sin \theta$$

CONVERT $(r, \theta) = (4, \frac{2\pi}{3})$ TO RECTANGULAR

$$x = 4 \cos \frac{2\pi}{3} = 4 \cdot (-\frac{1}{2}) = -2$$

$$y = 4 \sin \frac{2\pi}{3} = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$$

$$(x, y) = (-2, 2\sqrt{3})$$

USE EXACT VALUES
IF WORKING WITH
"COMMON" ANGLES

IE. MULTIPLES OF $\pi, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$

SANITY CHECK

① SIGNS OF x, y

$$x < 0, y > 0 \rightarrow Q_2$$

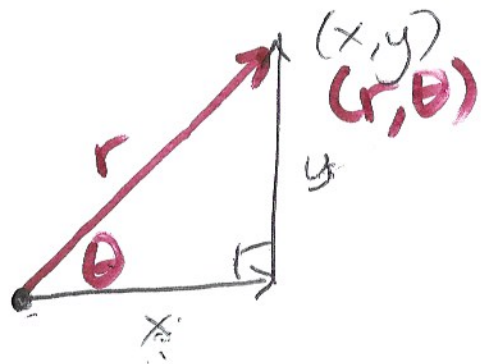
② SIZE OF x, y
↑
ABSOLUTE VALUE

$$|y| = |2\sqrt{3}| > |-2| = |x|$$

MORE
UP+DOWN

LESS
-LEFT+RIGHT

TO CONVERT RECTANGULAR (x, y) TO POLAR (r, θ)



GIVEN (x, y)

$$r^2 = x^2 + y^2 \rightarrow r = \sqrt{x^2 + y^2}$$

$$\tan \theta = \frac{y}{x}$$



$$\theta = \tan^{-1} \frac{y}{x}$$

?

?

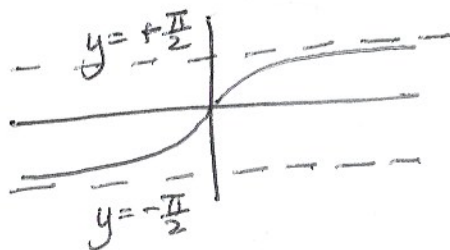
-

~~IF $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$ IF $x > 0$
BUT WE DON'T KNOW θ YET~~

IF $x < 0$

IF $x = 0$

RANGE OF $\tan^{-1} x$
IS $(-\frac{\pi}{2}, \frac{\pi}{2})$



$$\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$$

$$\theta \in Q_1 \text{ or } Q_4$$

$$\text{i.e. } x > 0$$